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Inheritance of Arithmetical Qualities; A Quantitative Study

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INHERITANCE OF ARITHMETICAL QUALITIES: A QUANTITATIVE STUDY

BY

MARGARET VARA COBB

A. B. Radcliffe College, 1910

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF ARTS

IN ZOOLOGY

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

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THE GRADUATE SCHOOL

June 4th 1913

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Margaret Vara Cobb

ENTITLED Inheritance of Arithmetical Qualities: A Quantitative Study

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Arts

Charles Zeleny
In Charge of Major Work

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Committee on Final Examination

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I wish to acknowledge very thankfully the cordial help I have received from many sources: from Dr. W. C. Bagley and Dr. L. D. Coffman of the Department of Education; from from Dr. M. Bentley of the Department of Psychology; Dr. H. L. Rietz of the Department of Mathematics; from the willing subjects of my experiments; and above all from Dr. Charles Zeleny, under whose supervision the work has been done, and whose constant criticism and suggestion have aided it more than I can say.

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Table of Contents.

I Statement of the Problem

II Method

III Materials

IV The Records

V Treatment of Records

VI Results

VII Sources of Error

VIII Appendices

IX Bibliography

Inheritance of Arithmetical Qualities:
A Quantitative Study.

I Statement of the Problem.

This paper is a study of the resemblance of child to parent in "arithmetical constitution"; that is, in the relations between the degrees of skill he shows in the simple arithmetical processes. It aims to show that if, for instance, a child is much quicker than the average in subtraction, but not in addition, multiplication or division, it is to be expected that one at least of his parents shows a like trait; or, if he falls below the average in subtraction and multiplication, and exceeds it in addition and division, again the same will hold true of at least one of his parents; it aims to show, further, that heredity is the most reasonable explanation of this resemblance. It is based on the results of a preliminary investigation begun in the fall of 1912 at the University of Illinois, where the coöperation of the Departments of Zoölogy and of Education gave unusual advantages for carrying on the work. It makes clear the possibility of testing the theory by actual measurements of families; it gives from the figures meanwhile obtained a provisional measure of the degree of resemblance in families, which is significantly high; and it points to the conclusion that further work, - which, it is hoped, may be undertaken in the near future, - will prove this to be a hereditary likeness.

II Method.

The measurements were made by means of the Courtis Standard Tests in Arithmetic. These tests, the work of Mr. S. A. Courtis of the Detroit Home and Day School, Detroit, Michigan, were devised to measure the ease and accuracy with which pupils in the grammar grades could handle two-figure combinations in addition, subtraction, multiplication and division, and a variety of simple problems in arithmetic. From averages based on the thousands of tests, Mr. Courtis set a standard in each operation for each grade, and built up from these a graph sheet on which the record of any child, or the average of a group, could be graphically shown. Of the eight tests, the first five - those from which these data are drawn - are described and illustrated in the following extract from Mr. Courtis' recent report (Courtis 1913) to the Committee on School Inquiry of the New York Board of Estimate and Apportionment, on the Courtis Tests in Arithmetic which he had just carried out in that city.

General Plan of Series

* * * * * *

"The tests are eight in number and are known as -

Test No. 1.	Speed Test	- Addition	Combinations 0 - 9
" " 2.	" "	- Subtraction	
" " 3.	" "	- Multiplication	
" " 4.	" "	- Division	
" " 5.	" "	- Copying Figures	(Rate of motor activity)
" " 6.	" "	- Reasoning	(Judgment of operation to be used in simple one-step problems.)

Test No. 7. Speed Test - Fundamentals (Abstract examples in the four operations - computation.)
" " 8. " " - Reasoning (Two-step problems involving computations.)

"Each of these is illustrated and discussed below. The reader will do well to get the general scheme clearly in mind in order to follow easily the discussion of results. He should note that tests 1 - 6 are the elemental abilities, tests 7 and 8 are complexes; that test No. 7 involves the use of the abilities tested in Nos. 1 - 5, and that No. 8 involves all the abilities measured in the previous test.

"The use of the term "Speed" in connection with the elemental tests needs explanation. The abilities themselves are so very elemental that if sufficient time were given practically every child above the fourth grade (except defectives) would be able to complete the whole of each of these tests, and to do the work correctly. Moreover, the attitudes of the children toward the work would be very different in the different grades by reason of changes in the maturity of the children. But by putting the work on a speed basis a situation is created that is much more nearly uniform from individual to individual, and from grade to grade - straight-ahead work at one's top speed - and the results disclose at once where weaknesses exist; for the abilities break down at their weakest points. Speed, then, is not insisted on for the sake of speed, but because speed best ministers to uniformity of conditions, and to exposure of weaknesses.

Details of Tests Nos. 1 - 4

" Taking up now the discussion of details, in figures

COURTIS STANDARD TESTS

"Measure the efficiency of the entire school, not the individual ability of the few"

SCORE
No. attempted .....
No. right .....

ARITHMETIC—Test No. 1. Speed Test—Addition

Name..... *School*..... *Grade*.....

Write on this paper, in the space between the lines, the answers to as many of these addition examples as possible in the time allowed.

1 6 9 0 4 2 6 5 1 2	1 7 9 3 2 3 7 6 0 4	1 3 6 0 3 5 8 9 7 2	8 9 7 8 2 1 9 6 0 5
1 4 8 0 2 6 7 9 5 7	3 4 7 0 3 1 6 9 8 5	1 2 5 6 7 4 9 8 0 2	5 8 6 9 4 1 3 5 0 3
2 9 7 4 5 2 3 8 0 2	1 3 8 2 3 7 9 5 0 7	4 8 9 5 3 1 8 7 0 6	1 8 6 0 5 9 4 7 2 4
7 4 8 0 3 1 9 6 0 4	9 2 5 0 6 1 8 7 4 3	2 4 5 1 6 1 8 9 0 2	3 7 9 0 4 3 4 8 6 5
4 8 5 0 7 4 2 6 9 3	5 9 6 7 5 5 2 8 0 3	1 6 7 0 2 8 4 5 3 6	6 9 8 1 2 1 4 7 1 3
1 7 9 3 2 3 7 6 0 4	8 9 7 8 5 1 9 6 0 2	1 2 6 0 3 6 7 9 7 2	1 4 9 0 4 6 7 5 1 2

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Figure I

COURTIS STANDARD TESTS

"Measure the efficiency of the entire school, not the individual ability of the few"

SCORE
No. attempted .....
No. right .....

ARITHMETIC—Test No. 2. Speed Test—Subtraction

Name ..... *School* ..... *Grade* .....

Write on this paper, in the space between the lines, the answers to as many of these subtraction examples as possible in the time allowed.

9 7 11 8 12	1 9 13 4 12	8 11 12 5 10	6 11 15 10 12
9 3 6 1 3	0 7 8 3 6	0 9 7 1 2	0 7 8 9 4
2 7 13 3 10	1 6 15 4 8	4 10 13 10 9	5 8 17 6 11
1 5 7 2 5	1 3 9 2 3	4 7 5 1 4	5 6 9 4 8
4 9 16 7 11	5 12 15 5 16	7 8 16 9 11	9 7 14 5 11
0 6 9 1 3	0 9 6 3 8	0 5 7 1 4	0 2 8 4 2
2 12 15 3 10	8 13 12 8 13	2 10 11 9 14	6 7 17 6 9
2 8 7 1 3	8 9 5 7 4	0 6 5 8 7	6 4 8 1 2
0 5 14 7 8	3 9 14 4 18	7 6 13 10 9	3 8 14 6 10
0 2 5 6 2	3 5 6 1 9	7 2 6 8 3	0 4 9 5 4
8 11 12 5 10	9 7 11 8 12	6 11 15 10 12	1 9 13 4 12
0 9 7 1 2	9 3 6 1 3	0 7 8 9 4	0 7 8 3 6

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Figure II

COURTIS STANDARD TESTS

"Measure the efficiency of the entire school, not the individual ability of the few"

SCORE
No. attempted .....
No. right .....

ARITHMETIC—Test No. 3. Speed Test—Multiplication

Name..... *School*..... *Grade*.....

Write on this paper, in the space between the lines, the answers to as many of these multiplication examples as possible in the time allowed.

2 3 9 0 7	9 5 4 7 6	4 2 7 4 9	3 4 9 0 5
1 3 6 5 4	1 2 8 0 5	1 9 6 0 5	2 7 8 2 6
8 2 7 5 4	2 5 6 0 7	1 2 7 0 8	1 2 8 1 5
1 6 9 0 6	3 5 9 8 3	6 8 7 6 3	9 5 7 1 3
6 2 8 9 5	4 3 9 8 6	1 3 7 6 5	3 2 6 0 8
1 7 4 0 7	2 6 7 0 4	2 5 8 0 9	1 4 7 1 5
1 3 6 0 3	1 6 8 0 9	1 4 8 0 4	7 3 9 2 4
7 4 8 0 9	4 2 8 7 3	5 4 9 3 5	1 8 9 0 3
5 8 6 0 5	1 9 8 0 3	2 5 4 3 7	1 6 9 1 7
1 2 3 9 4	3 2 6 4 7	2 8 9 0 5	8 2 4 0 2
9 5 4 7 6	2 3 9 0 7	3 4 9 0 5	4 2 7 4 9
1 2 8 0 5	1 3 6 5 4	2 7 8 2 6	1 9 6 0 5

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Figure III

COURTIS STANDARD TESTS

"Measure the efficiency of the entire school, not the individual ability of the few"

SCORE
No. attempted _____
No. right _____

ARITHMETIC—Test No. 4. Speed Test—Division

Name..... *School*..... *Grade*.....

Write on this paper, in the space between the lines, the answers to as many of these division examples as possible in time allowed.

3)9	4)32	6)36	2)0	7)28	9)9	3)21	6)48	1)1	5)10	2)6	4)24	7)63	6)0	8)32	1)8	5)30	8)72	1)0	9)36
8)8	4)28	5)40	2)2	8)16	1)4	5)35	9)45	2)2	3)12	1)5	2)16	8)48	1)2	9)27	5)5	4)36	9)54	8)0	4)12
1)7	2)10	7)42	1)1	6)18	3)6	4)20	7)49	1)3	2)8	6)6	3)27	8)64	1)2	4)16	5)0	3)24	9)63	2)4	8)24
5)5	2)14	8)40	9)0	5)15	7)7	2)18	6)42	3)0	7)21	4)4	6)30	8)56	1)0	7)14	4)4	3)15	9)81	7)0	6)12
1)4	2)12	5)45	3)3	4)8	1)5	3)18	9)72	4)0	6)24	1)9	5)25	7)56	3)3	9)18	1)6	7)35	6)54	1)3	5)20
9)9	3)21	6)48	1)1	5)10	1)8	5)30	8)72	1)6	9)36	2)6	4)24	7)35	6)0	8)32	3)9	4)32	6)36	2)0	7)28

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Figure IV

"Measure the efficiency of the entire school, not the individual ability of the few"

SCORE
No. attempted _____
No. right _____

ARITHMETIC—Test No. 5. Speed Test—Copying Figures

Name ..... *School* ..... *Grade* .....

Copy on this paper, in the space between the lines, as many of the printed figures as possible in the time allowed. Write as rapidly as possible, but form the figures as carefully as in working examples.

2 4 9 6 7 4 2 9 7 6 6 2 9 4 7 7 2 9 6 4 2 4 9 7 6 4 2 9 6 7 6 2 9 7 4 7 2 9 4 6

2 6 9 7 4 4 6 9 2 7 6 4 9 7 2 7 4 9 2 6 2 6 9 4 7 4 6 9 7 2 6 4 9 2 7 7 4 9 6 2

2 7 9 4 6 4 7 9 6 2 6 7 9 2 4 7 6 9 4 2 2 7 9 6 4 4 7 9 2 6 6 7 9 4 2 7 6 9 2 4

2 4 9 6 7 4 2 9 7 6 6 2 9 4 7 7 2 9 6 4 2 4 9 7 6 4 2 9 6 7 6 2 9 7 4 7 2 9 4 6

2 6 9 7 4 4 6 9 2 7 6 4 9 7 2 7 4 9 2 6 2 6 9 4 7 4 6 9 7 2 6 4 9 2 7 7 4 9 6 2

2 4 9 6 7 4 2 9 7 6 6 2 9 4 7 7 2 9 6 4 2 4 9 7 6 4 2 9 6 7 6 2 9 7 4 7 2 9 4 6

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Figure V

"Measure the efficiency of the entire school, not the individual ability of the few"

COURTIS STANDARD TESTS

Individual Score Sheet—Arithmetic

SCORE	
No. attempted	
No. right	

Name....., ... Age last birthday..... Grade..... Room.....
(Boy or Girl)

School..... City..... State..... Year 191.....

Date: Set No. 1...../...../...../..... Set No. 2...../...../...../.....
(Day of week) (Month) (Day of month) (Hour) (Day of week) (Month) (Day of month) (Hour)

Test No.	Subject	First Trial	Second Trial	Change	Tests No.	Subject	Examples Attempted			Examples Right		
							First Trial	Second Trial	Change	First Trial	Second Trial	Change
	Month. . . .					Month						
1.	Addition . .				5.	Copying figures						
2.	Subtraction .				6.	Speed reasoning						
3.	Multiplication				7.	Fundamentals .						
4.	Division. . .				8.	Reasoning . . .						
Total	Tests 1 to 4											

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Figure VI

1, 2, 3, and 4 will be found illustrations of the first four speed tests. As these differ merely in the operation to be used, the characteristics of all four can be discussed at one time. These are (1) completeness; and (2) uniformity.

1. Completeness

"Each test contains in its first five lines the 100 fundamental combinations for its operation (except division). The sixth line is a repeat in order that, when necessary, the entire hundred combinations may be finished without having the work influenced by the knowledge that the end has been reached. It will be noted that the expression "one hundred fundamental combinations" is used in place of the conventional "forty-five fundamental combinations" and their reverses. Analysis of the mistakes of many individuals as well as direct experiments in the laboratory have proved conclusively that it is possible for an individual to know the result of three times four without knowing that of four times three; of four plus three, and not that of three plus four. Emphasis on the number of combinations, as 45, tends to restrict the teaching to the direct combinations only. Careful time studies in the laboratory make it certain that each individual's action toward each combination is highly specialized. Consequently, it may become important to test an individual's response to every one of the hundred combinations in use in order that, where peculiarities exist, they may be detected and special remedial work undertaken to remove difficulties.

"Many educators declare, also, that the zero combinations should not be included in such a test. This is, of course,

a matter of opinion. The inclusion of such combinations in these tests, however, is not an arbitrary decision, but a deliberate judgment based upon the results of many tests and experiments. For instance, (1) it was found that in the papers from the Stone tests, where a cipher occurred in the multiplier, many mistakes were made in the failure to multiply by this figure correctly; (2) that in a multiplication speed test of forty-one mature individuals - graduate students, teachers, superintendents, etc. - at Columbia University, in the summer of 1910, eighteen of the group made 104 mistakes in the zero combinations; (3) that in classes in high-school algebra substitution involving the use of zero as a multiplier causes many mistakes; (4) that children taught the zero combinations in the regular way in the early grades have no difficulty with them, while classes which have never received this training not only make mistakes, but mistakes persist if remedial training is attempted later than the seventh grade; finally (5) that, even when testing adults with sufficient control to write the answer correctly, time studies show the lengthening of the interval required to make the proper response to these combinations. For all these reasons, and for others which cannot be discussed here, the entire one hundred combinations are included in the tests.

"A single exception is made in the case of the division speed test. Here the combinations - eight into zero, six into zero, etc. - are included, but the forms - zero into eight , etc. - are not. The answer in these cases requires the use of a symbol with which the children are not familiar, and the form itself is never met in elementary mathematics. They have been replaced by

repetitions of the simpler combinations.

2. Uniformity

"The distribution of the combinations within a test is not a chance one, but is based upon a regular plan. As shown in Table I, the one hundred combinations are grouped into five equal divisions differing in degree of difficulty. As previous tests had proved that to any one individual the actual difficulty of the various combinations was an individual peculiarity - that to one person seven times eight might present wellnigh insurmountable difficulties - the basis for estimation of degree of difficulty was taken as the effort that would be involved in working out the answer if it were not remembered. On this basis of five divisions, shown in Table I , were formed; one combination was chosen from each division, each group of five combinations in the test being thus the equal of any other group. In making up the groups, several other factors were taken into consideration, namely, the number and difficulty (since some figures take longer to write than others) of the figures that must be written in each line in filling in the answers, and the distributions of the various tables throughout the tests, so that the combinations of any one table might not occur too closely together.

Table I - Showing Subdivisions of the Combinations in Test I

GROUP I - Very Easy	0 0 0 0 0 0 0 0 0 0 0 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 0 0 0 0 0 0 0 0 0 1
GROUP II - Easy	1 1 1 1 1 1 1 1 2 2 2 3 4 5 6 7 8 9 1 2

GROUP II - Easy (Continued)

3 4 5 6 7 8 9 3 4 5
1 1 1 1 1 1 1 3 4 5

GROUP III - Average

2 2 2 2 2 3 3 3 3 3
3 4 5 6 7 2 4 5 6 7

4 4 4 5 5 5 6 6 7 7
2 3 5 2 3 4 2 3 2 3

GROUP IV - Hard

2 2 3 3 4 4 4 4 6 6
8 9 8 9 6 7 8 9 4 6

7 8 8 8 9 9 9 7 8 9
4 2 3 4 2 3 4 7 8 9

GROUP V - Very Hard

5 5 5 5 6 6 6 6 7 7
6 7 8 9 5 7 8 9 5 6

7 7 8 8 8 8 9 9 9 9
8 9 5 6 7 9 5 6 7 8

"In any such scheme involving the interplay of several factors, compromise is inevitable, but the resulting tests - the end products of a long series of rearrangements - have proved remarkably uniform."

* * * * *

Speed Test No. 5; Copying Figures.

"The purpose of this test is to determine the rate of motor activity, not how well or how poorly children can copy figures. The importance of this knowledge is shown by the following: Two girls in the same class obtain scores in, say, the subtraction test which are just half the value of the class average. In a test of their rates of copying figures, one equals the average rate of the class, but the second again makes a score half the class average. It is evident that the first girl failed in the subtraction test because she did not know the combinations. The second, however, even if she knew her combinations perfectly,

could not have equaled the average score of the class because she was handicapped by her inability to make her fingers move at the average rate. For corrective work, the difference is fundamental.

"It will be noted that the work of this test calls for a minimum of mental activity. Practically nothing is demanded except perception of the figures to be written, and the proper motor responses. As, however, the adult score is from 110 - 130 figures per minute, close and sustained attention must be given to the work. The results measure quite accurately the relative native ability of the children. Other things being equal, that individual who is able to expend energy at the greater rate has the greater ability. Also, in many cases, the results are an indication of the stage of development reached by the children, particularly in

the lower grades. Scores much below, or in excess of, the grade average are often indications of retarded or advanced physical development.

"Uniformity in this test was secured by repeated use of the same figures. Many experimental determinations of relative difficulty, based upon the time required to write different figures, were made. For instance, it was found that, on the average, a person can write three figure 1's while he is writing one figure 5. The figures 2, 4, 6, 7 were selected as being the four of nearest average value; the 9 as a more difficult figure. In the different groups in the test, the 9 is always placed as the central figure of the five, in the belief that a rhythm might be established which would tend toward uniformity. About the 9 the other figures are grouped in every possible variation. The test as a whole has yielded consistent and satisfactory results.

* * * * * *

Order in Which Tests are Given.

"The tests were printed from plates, and bound by a small staple in the upper left-hand corner into sets as follows: Set 1 - Record Sheet, Tests 5, 2, 8, 3; Set 2 - Tests 4, 6, 1, 7. The order is that which experience has proved best. Test 5 was least influenced by the long exposure during the first giving of instructions. Test 2 was chosen to precede Test 8 as having the least influence upon it. Test 8 was given before the remaining tests that its results might not be influenced by them; Test 7, the other complex, was put last that it might be influenced by

all alike. Of the remaining tests, No. 1 was put as late as possible because many children, having once added, find difficulty in immediately undertaking other speed tests."

Mr. Courtis describes the tests from an educational point of view, but his account brings out their accuracy, and their fitness for the work of comparing abilities in the same individual, and individuals with one another.

In the work of measuring families, the tests were given almost as Mr. Courtis describes them. The time allowance for each of the five was one minute; the results of the three longer tests made at the same time will be given in a later paper. To lessen the chance effect of time, place, and condition of the individual, each test was repeated before going on to the next and the average of the two, later, taken as the score. Uniformity in starting was furthered by asking each person to turn his sheet face downwards, hold it with the left hand by the lower right hand corner, and turn it over when the signal for the start should be given. This signal was the word "Now", preceded by about two seconds by the warning word "Ready". The signal for stopping was the word "Stop". For the timing a stop-watch was used; the inaccuracies in timing were slight and entirely negligible, being probably the least of the factors of error in the work. Directions to each group were made as brief and simple as possible, - and as uniform. To this end I always gave them myself, explaining merely that the object of the test was to learn each person's comparative speed in the different kinds of examples, and pointing out how this made it necessary to do each

as fast as he could. I tried to have each group, and each member of a group, take the test in the same spirit, but differences in temperament make this well-nigh impossible. This however is a factor which presumably affects the tests as a whole and not relative excellence in any two tests. The resemblences found cannot be due to the fact that all the members of a family were tested at the same time, for (1) an unrelated person tested with a family does not show the resemblence, and (2) a group of unrelated persons tested together does not show resemblence among its members.

Between each two tests there was an interval of two or three minutes, during which each subject was asked to count his answers and mark the score on the sheet itself and on the record sheet he had already filled out with his name, sex, age, and the date and time of the test. These counts were later verified; at this time they servedthree purposes - (1) they added interest to the tests (2) they took up the time of relaxation between the tests (3) they made a single rechecking sufficient.

The time of day at which such tests are made, and the state of fatigue of the subject, unless extreme, according to the latest studies play almost no part in the result (Heck 1913,b pg 33). Outward conditions, however, which could not be kept quite uniform, such as convenience of height of table, disturbances, etc., perhaps have more effect. These, and the differences due to the physical condition of the subject, which were for the most part unknown and uncontrolled, seemed to have caused what differences were found when the same person was tested at

different times. These were not enough to change materially the direction of the curve. Figure VII shows the curves of a person tested four times, at nine-day intervals. More permanent factors of the same kind, such for example as the acquired left handedness of a man who had had writer's cramp, probably had similar effects which could not be observed. It seems probable that in many such cases all the scores are lowered, changing the position of the curve more than its shape. It is suggested that this may be a reason why children and parents are more alike in the relations they show between the various abilities than in absolute standing in any one ability.

What differences may be due to age is unknown, so no steps could be taken to eliminate them. It seems probable that there may be a slowing down of all processes, after maturity is passed; in this case there need be no change in the shape of the curve. In face of the close likenesses between parent and child this conclusion is almost forced upon us.

No sex differences have been noted. This accords with Mr. Courtis' observations on thousands of cases. Moreover, a child of either sex may resemble either parent; and all the children of the same family do not always resemble the same parent.

In such simple tests the percentage of wrong answers is very small and rather constant. Speed alone, therefore, may be taken as a measure of skill. Mr. Courtis says "It will be remembered that in Tests Nos.1 to 5 the scores represent the number of answers written per minute; in Tests 6, 7, and 8 the number of examples attempted and the number right. No scoring of rights in Tests 1 to 4 was made, as experience has shown that

the mistakes, except in the zero combinations in multiplication and the zero and one combinations in division, are too few in number to pay for the labor expended. The sight of two figures in position for addition, for instance, serves as a stimulus to which the habitual response must be made, if any. That is, a child that does not know his tables is apparently unable to make up and write any answer. Caught in the grip of a situation to which he has previously made response he apparently must go through some habitual process of recall before he can even write an incorrect answer. In fact, so powerful is the grip of habit that most children persist until the right answer is recalled, ignorance of the tables being shown, not by incorrect answers, but by low scores."

Thus the score consists of the number of answers written, not the number correctly written, at the end of one minute. The counting and recording of the scores has all been carefully checked, and is believed to be accurate.

III Material

To find families willing and able to give an hour or an hour and a half of their concerted time to the taking of these tests is no easy matter. A beginning was made by sending a circular letter to eighty members of the faculty of the University of Illinois, inquiring as to their willingness to take part in the tests. From the twelve families who replied favorably, and almost as many others learned of from these, careful records have been gathered. It is on certain of these, - all, in fact, in which the children were over fourteen years, - that this preliminary report is based. To make possible the comparison of these records, however, it was necessary to know the average adult scores for the class of people under consideration. To this end several university classes, in education and zoölogy, were through the coöperation of Dr. Bagley and Dr. Zeleny submitted to the tests, as was a volunteer group of graduate students in science. It will be noticed that all the work has been done among a highly educated class of people. It seemed most feasible, and also advisable, that the problem should be thus limited at first, in order that the results of the preliminary work should have as definite a meaning as might be. All the averages, therefore, and exact numerical statements will be understood to apply merely to this class; it would seem that the generalizations, however, might fairly be extended to all classes of people.

IV The Records

The records from which conclusions are here drawn are the score sheets of one hundred ten unrelated adults, and of eight families (See Appendix I). Since a method of comparing children who are still learning, with adults, has not been perfected, only those of over fourteen years are included. Twenty such records were secured. Each score sheet gives the score in each of the two trials of each test, with the average of the two. In two families four trials on different days were made, in the hope of increasing the accuracy. These were in one case at four-day, in the other at nine-day intervals, that the effect of practice might be kept down(See Figure VII). Seven individuals also who had taken the tests in the ordinary way were tested a second time after an interval of weeks, and one a third time; as the second and third curves were similar in form to the first, the chance errors in making the tests in this way may probably be neglected. For the mathematical probability is that such errors will tend to hide whatever likeness there is between child and parent, and it may safely be said that correlation,where high, is greater than appears.

V Treatment of Records

Very early in the work, as has been said, the need for an adult average was felt. To meet it, as many adults as possible were tested during the year, and from the first one hundred ten records the averages used in calculation were computed. (See Table I). Later records served as a chedk on these. Finally, from these and the standard deviation in each operation a graph

	Average	Standard Deviation
Addition	75	14
Subtraction	62	13
Multiplication	54	10
Division	60	14
Copying Figures	126	16

Table I. Average and Standard Deviation for each operation.

sheet has been arranged which seems suitable for use with individuals over fourteen years. The average for each operation forms the zero point of the scale, above or below which every individual falls; these zero points are placed one below another down the middle of the sheet. The standard deviation is taken as the unit distance above and below zero. Thus a person whose score in addition is above the average by two and one tenth times the standard deviation stands at +2.1 on the line representing addition. (See Figure VII)

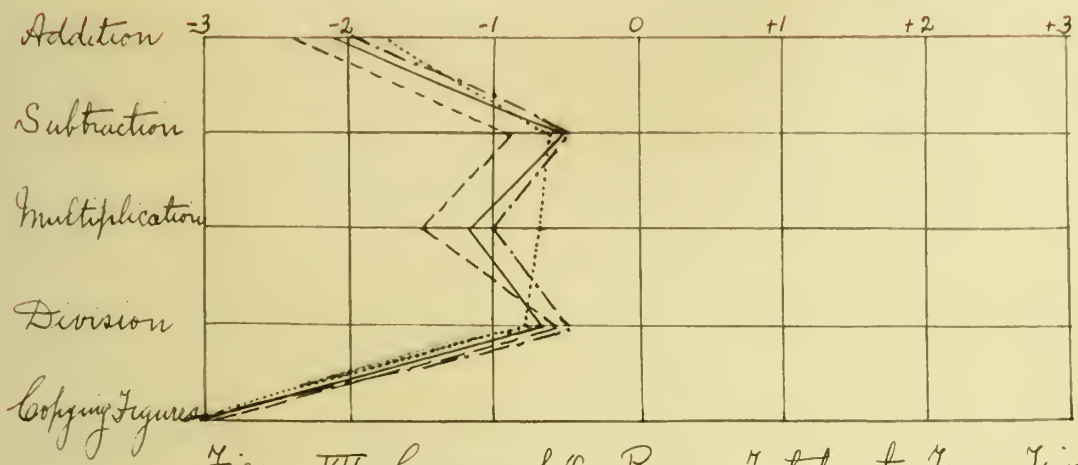

Figure VII Curves of One Person Tested at Four Times.

1st Test ———

2nd Test .....

3rd Test – – – –

4th Test —.—.—

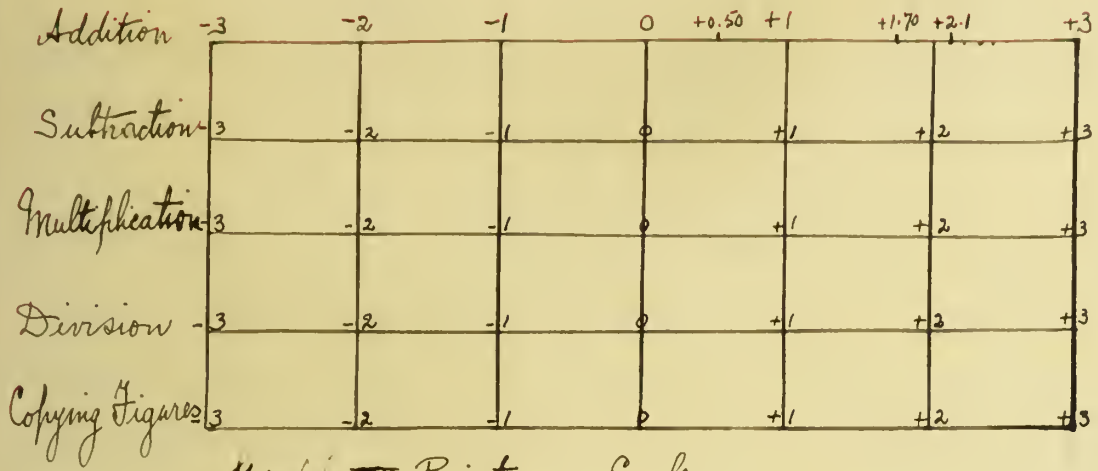

Figure VIII Points on Scale.

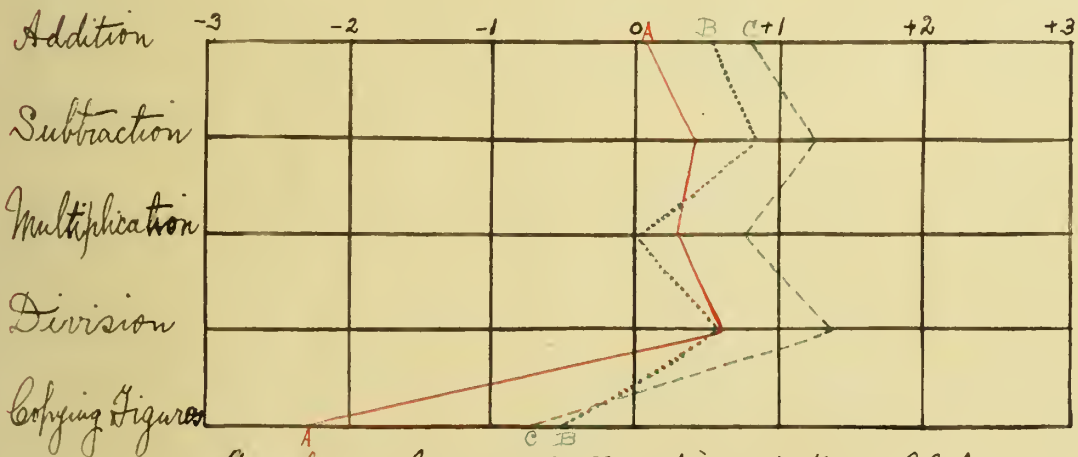

Figure IX. Curves of Parent and Two Children.

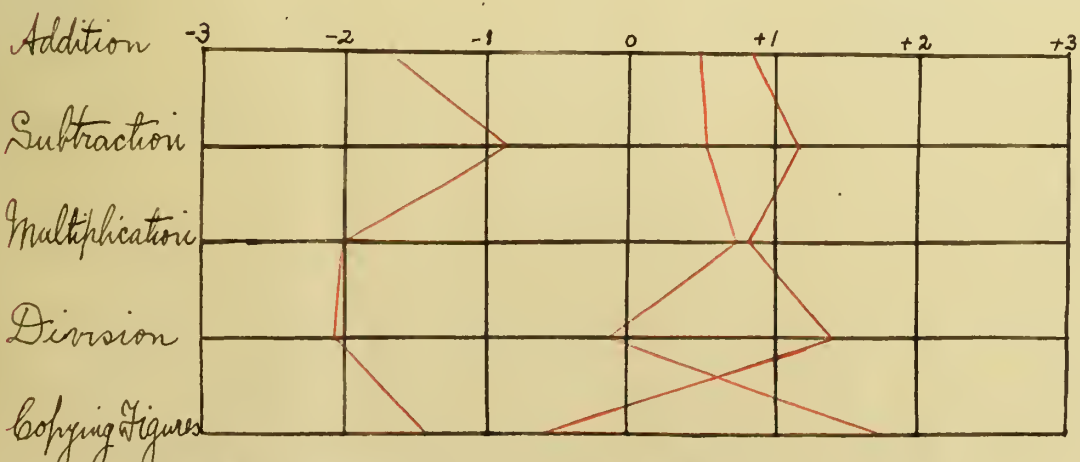

Figure X. Three Unrelated Curves.

In the numerical handling of the records, also, individual standing has been expressed in terms of the standard deviation unit; that is, as being so many times the standard deviation, above or below the average. For instance, suppose a score in addition is 82 examples done in one minute. This is seven more than the average. Since the standard deviation is 14, this score would become +0.50, or half a unit above the average. (See Figure VIII) Or, a score represented as +1.70 would be interpreted thus: since +1.70 means above the average by 1.70 times the standard deviation, which is 14, this score is 24 above the average, which is 75. This means an actual accomplishment of 99 examples in one minute. (See Figure VIII).

This method of expression was chosen in order to bring all the operations to the same terms, that they might be directly compared. If, for instance, my score in addition is +0.55, and in subtraction +0.86, the relation between them may then be expressed as +0.31 (of the standard deviation unit, in each case); that is, to move from the addition standing to the subtraction standing is to move in the positive direction through .31 of the standard deviation unit, which is made the same size in each case. A glance at Figure IX, curve B, will make this clear. Further, if the multiplication score is +0.03, the division +0.63, and copying figures -0.50, the subtraction - multiplication relation is -0.83, the multiplication - division relation +0.60, and that between division and copying figures -1.13. (See Table II and Figure IX, Curve B) It is relations such as these which have been compared in parent and child, and found on the whole to show striking

likeness. If, for instance, my mother's scores, tho differing in absolute value from mine, still bear to one another the relations shown in Table II and Figure IX, Curve A, there is evident

Relations	Mother	1st Daughter	2nd Daughter
Add. - Sub.	+0.35	+0.31	+0.36
Sub. - Mul.	-0.10	-0.83	-0.44
Mul. - Div.	+0.36	+0.60	+0.57
Div. - c.f.	-2.91	-1.13	-2.08

Table II Relations between Operations in Mother and Daughters.

some degree of correspondence, while with the figures given for a younger daughter the correspondence is even closer (See Table II and Figure IX, Curve C). Figure X shows the curves of three unrelated persons. In calculating the coefficients of correlation, not only these successive relations, but also the relations between addition and multiplication, addition and division, subtraction and division, and between each of these and copying figures, have been used. The degree of correspondence has been worked out by the use of the following formula,

$$r = \frac{\Sigma x \cdot y}{n \sigma_1 \sigma_2}.$$

in which r gives the Pearson coefficient of correlation; the numerator is the sum of the products of the measurement of the trait in the parent by the measurement of the same in the child; and the denominator is the product of the number of cases by

the standard deviation of the measurements of the traits in parent by the standard deviation of the measurements of the traits in child.

IV Results.

Before giving the results of the work, I wish to bring up once again the point, that the correlation between the actual scores of child and parent in a given operation is not the subject of this paper. That such a correlation exists, the curves bear witness; and some attention has been given to it. A child whose parents make high scores is more likely than not to make high scores himself; in addition, for instance, which has been fully worked out, the coefficient of correlation with the more like parent is +0.30. But a child whose parents make better scores in subtraction than in addition is even more likely himself to make a better score in subtraction than in addition; here the coefficient of correlation with the more like parent is +0.56. In terms of the graph, it is likeness in shape, not in position, in the curves of parent and child with which we are here concerned. (See Figure IX)

The simplest way to get the significance of the correlations which show themselves between parent and offspring from this treatment of the data is to ask ourselves what the figures would be, under given conditions; and to use these as a basis of comparison. Three hypothesis are possible: (1) there is no resemblance between these traits in child and in parent; (2) there is resemblance due to the action of heredity; (3) there is resemblance due to the action of environment. First, if a child showed no tendency to do better, in the operations in which his parents did better, than in other operations, the following statements should hold true:

Child's - - - - Coefficient of Correlation				
with	With no resemblance	With resemblance due to		Actual figures
		Inheritance and blending	Inheritance and segregation	
Midparental standing	0.00	Almost +1.00	Over +0.50	+0.49
More like parent	Slightly positive	Positive	+1.00	+0.60
More unlike parent	Slightly negative	Slightly less positive	0.00	+0.01

Table III. Coefficients of Correlation.

Child with	Simple operations	Copying figures	Average
Midparental standing	+0.36	+0.61	+0.49
Like parent	+0.45	+0.78	+0.61
Unlike parent	+0.04	+0.24	+0.14

Table IV. Coefficients of Correlation. Method 1.

Child with	Simple operations	Copying figures	Average
Midparental standing	+0.34	+0.65	+0.49
Like parent	+0.48	+0.71	+0.60
Unlike parent	-0.12	+0.15	+0.01

Table V. Coefficients of Correlation. Method 2.

1. Between child and an arbitrary midparental standard computed from the records of the two parents (See Appendix II), there should be no correlation, that is, the coefficient should be zero.

2. Between child and the more like parent there should in the long run be a small positive correlation, because the comparison is made each time after choosing the more like of two parents.

3. Between child and the more unlike parent there should in the long run be a small negative correlation, because the comparison is made each time after choosing the more unlike of two parents.

A glance at Table III, which compares the figures actually found with this and the following hypotheses, shows that this one at least is far from fitting the facts. The figures indicate, on the contrary, a strong resemblance to one parent, and a distinct resemblance to the midparental standing.

If, in the second place, the resemblance is due to heredity alone, there are two extreme cases for which we can foretell the correlations rather exactly. If, on the one hand, the qualities in the child are the result of a blending of those of the parents, the conditions should be these:

1. Between child and midparental standard there should be perfect correlation, diminished by the tendency to regress towards the mean. The coefficient should be less than +1.00.

2 and 3. Between child and each parent there should be a positive correlation, very much less than one, and very little greater when the like than when the unlike parent is chosen.

Again ,this does not fit the actual figures.

If on the other hand the relations between operations are inherited by each child from one parent only, (that is, if there is segregation of these qualities), the following should be the conditions:

1. Between child and midparental standing there should be a positive correlation greater than +0.50.

2. Between child and like parent there should be perfect correlation; that is, the coefficient should be +1.00.

3. Between child and unlike parent there should be no correlation; that is, the coefficient should be zero.

Table III shows that the figures found come closer to this last condition than is usually found in working on the inheritance of human traits, where heredity correlations are bound to be obscured from many sources. So this hypothesis may at least be provisionally retained; and, since the figures fit this and not the previous hypothesis, it must be said that if the resemblance is due to inheritance, there is partial if not complete segregation in the germ cells of the factors involved.

If, in the third place, the resemblance is due to environment alone, we can make no forecast as to the relative values of these three correlations. We must look at the question from the other side; given these resemblances, can we find factors in the environment which may have produced them? Are there relations between the child and his world which can have acted in such a way as to bring about a strong likeness in these traits

to one parent, and practically none to the other? If a child is associated more closely with, or directly taught by, one parent, will he grow to resemble that one in the comparative ease with which he adds and subtracts? If so, a child should become like his teacher. He does not, for Mr. Courtis has shown that there is a wide range of varibility among children who have been under the same teachers and in the same classes, - far wider than the range among averages of children of the same grade, in different schools. If the curves of the children owe their shape to association with the teacher, the reverse should be the case. But if the training received from the teacher does not bring this about, it does not seem that association with one parent could effect it, especially when the figures show that the other parent, with whom in many cases he has been almost as closely associated, has had no influence. This is a question on which further tests may throw more light; but if, in view of the approximation of the figures to what we might expect if segregating inheritance were the basis of the likeness, we provisionally accept this explanation, then it must be considered whether training does not introduce a large factor of error in studying the laws of that inheritance.

The coefficients of correlation for the relation of speed in copying figures to speed in the simple processes, in child and in parent, are higher than those for the relations between the simple operations themselves. This feature is constant throughout the results. Two possible reasons for this have come up, but these are suggestions merely. The higher correlation may be dependent on the fact that the copying figures

test has a psychological basis somewhat different from the others; it involves fewer factors and requires no "mathematics" whatever. A somewhat more likely suggestion follows from the fact that in five of the eight families, including seventeen of the twenty children, the scores of the like parents happened to be extremely low in copying figures. Where the parent's variation from the average is so extreme, it may be that the trait is more strongly inherited; and it certainly seems evident that environmental or accidental differences in the curves, which in cases nearer the average might be so proportionally large as to hide the resemblance, would here have to be enormous to do so. Further tests should determine this; if it be true, it is direct evidence that accidental and environmental differences have played a part in lowering the other correlations.

VII Factors of Error.

The factors of error, tho many, cannot be held to be responsible for the character of the results. It was clear from the curves as they were first plotted on Mr. Courtis' graph sheet that such numerical results must appear. Nevertheless the sources of error should be carefully considered; they show that stress must be laid not on the exact figures, but on the general trend of the results. In general, where there is a high correlation, the effect of chance errors is to lower the coefficient.

The factors of error may be considered in four groups. 1. Those due to method, which have been eliminated as far as possible, and were discussed under method. 2. Those due to the nature of the material, some of which could not be eliminated. These also were discussed under method. 3. Those due to the small number of cases studied. 4. Difficulties in interpretation of results.

3. The mathematical means which were taken to make the scores comparable, and method by which a coefficient of correlation was found, were worked out by Dr. Zeleny and myself. Correlations between unrelated individuals, where the coefficient is zero, were secured by using the same methods. The data however are few. When the related pairs (parent and child) are grouped according to the particular trait considered, there are twenty such pairs in each group; as ten traits are considered, the figures for the average of the ten are based on 200 pairs. (Table IV) When the related pairs are placed in groups each consisting of ten traits in one child and its parent, there are ten in each group, but twenty groups. (Table V) The figures

for the average of the groups, based on the same 200 pairs, differ from the others by a small percent only. Tables IV and V give the figures thus obtained. (See also Appendices III and IV)

4. In the interpretation, of course the main difficulty is to distinguish between the effects of heredity and those of environment. Careful consideration of the results has led to the conclusion that heredity is probably the basis of the likeness found, and that in studying this, environment, chiefly in the effects of training, is to be looked to as the main factor of error.

Summary.

1. This report is based on a quantitative study of the resemblance in arithmetical constitution between twenty children and their parents. Since this included all the records of persons over fourteen years for whose parents records could be obtained, it is not a selected group.

2. Children resemble their parents in arithmetical constitution. That is, the relations of a child's skills in the four simple processes to each other and to his speed in copying figures tend to be like the corresponding relations in one of his parents. His skill in one process tends to be greater or less than his skill in any other, according as the same skill in the parent is greater or less than that of the parent in the other.

3. A child of either sex may bear this resemblance to either parent, but to one only. Pearson's correlation formula gives as the coefficients for these twenty children +0.60 when compared with the like, and +0.01 when compared with the unlike parent.

4. A child shows a greater resemblance to the like parent than to a standard taken between the records of the two parents. The Pearson coefficients are +0.60 and +0.49 .

5. Many considerations point to the conclusion that this is a hereditary likeness.

Appendix I

Average Scores of Sixteen Parents and Twenty Children (Eight Families) Used in Calculating Coefficients of Correlation.

Family		Age	Addition	Subtraction	Multiplication	Division	Copying figures
1.	Father	52	72.5	59.5	55.5	59.5	115
	Mother	57	76	67.5	57.5	69.5	90
	Daughter	27	83	73.5	54.5	69	118
	Son	26	87	70	61.5	65.5	119
	Daughter	22	70	62	56.5	50.5	113.5
	Daughter	20	87.5	78.5	62.5	79	114.5
	Daughter	19	73	62.5	49.5	61	114.5
2.	Father	51	72.5	63	49	58	89.5
	Mother	51	66	51	23	42.5	106.5
	Son	23	100	82.5	57.5	78	143
	Son	21	70.5	57.5	47.5	42	98
	Daughter	19	69.5	56.5	49.5	60.5	113
	Son	17	66.5	52.5	39	50.5	108.5
	Son	16	61	57	35.5	53.5	102
3.	Father	62	51	49.5	33	33	102
	Mother	55	36	24	27	25	86.5
	Daughter	28	65.5	56.5	49	55	121
	Daughter	25	74.5	61	47	50	115
	Daughter	22	38	43	35	30	101.5
4.	Father	—	77.5	63.5	52.5	61	101
	Mother	42	66.5	59.5	49.5	54	133
	Daughter	18	76	58	48	60	118
5.	Father	44	62.5	50	47	35	133.5
	Mother	—	82	69	61.5	59	154.5
	Son	15	56	29.5	46	50	139
6.	Father	42	87	66.5	55	62.5	126
	Mother	—	85.5	62	62.5	76	126
	Daughter	15	75.5	51.5	50.5	63	128
7.	Father	61	43.5	37.5	28	29.5	55.5
	Mother	52	67	49.5	49	53.5	115
	Daughter	25	108	111.5	72.5	107	137.5
	Son	18	60	40	41.5	39.5	127.5
8.	Father	44	81	67	57	65	135.5
	Mother	50	51.5	50.5	41	44	90
	Son	17	46	53.5	43	51.5	77
	Son	14	63	52.5	47	44	90

Appendix II

Meaning of Midparental Standard.

The midparental standard used in this work was obtained, not by averaging the actual scores of the two parents, but by averaging their curves; that is, by taking the figures which represented the directions of a curve lying midway between the curves of the two parents. The shape of this curve (and, incidentally its position also) is influenced equally by those of the two parents; and since it is curves which are being compared, and xxx figures representing their directions which are being correlated, it seemed best to take this mid-curve as the midparental standard.

Appendix III

Details of Table IV in Text. Correlation Figures Obtained by First Method. See Text pg. 36.

Relations	Correlation of child with midparental standard	Correlation of child with like parent	Correlation of child with unlike parent
Add.-Sub.	+.42	+.56	-.12
Sub.-Mul.	+.55	+.61	+.21
Mul.-Div.	+.39	+.49	+.18
Add.-Mul.	+.63	+.73	+.33
Add.-Div.	-.01	+.07	-.10
Sub.-Div.	+.18	+.22	-.24
Average	+.36	+.45	+.04
Add.-c.f.	+.68	+.77	+.33
Sub.-c.f.	+.61	+.77	+.23
Mul.-c.f.	+.51	+.86	+.23
Div.-c.f.	+.66	+.71	+.18
Average	+.61	+.78	+.24
Average of the two above averages.	+.49	+.61	+.14

Appendix IV

Details of Table V in Text. Correlation Figures Obtained by Second Method. See Text pg. 36.

	Correlation of Child with midparental standard in			Correlation of child with like parent in			Correlation of child with unlike parent in		
Case	Relations between simple operations	Relations of simple operations to copying figures	Averages	Relations between simple operations	Relations of simple operations to copying figures	Averages	Relations between simple operations	Relations of simple operations to copying figures	Averages
1	+.02	+.96	+.49	+.25	+.96	+.60	-.44	+.92	+.24
2	-.92	+.98	+.03	-.99	+.97	-.01	-.63	+.98	+.17
3	-.22	+.82	+.30	+.10	+.81	+.45	-.44	+.82	+.19
4	+.62	+.92	+.77	+.82	+.82	+.82	+.23	+.97	+.60
5	+.26	+.97	+.61	+.52	+.97	+.74	-.19	+.96	+.38
6	+.97	+.72	+.84	+.99	+.89	+.94	+.83	+.35	+.59
7	+.27	+.49	+.38	+.22	+.94	+.58	+.27	-.27	+.00
8	+.15	+.61	+.38	+.13	+.94	+.53	+.16	-.13	+.01
9	+.96	+.88	+.92	+.97	+.69	+.83	+.83	+.54	+.68
10	+.86	+.92	+.89	+.93	+.80	+.86	+.85	+.43	+.64
11	+.25	+.74	+.49	+.19	+.40	+.29	+.03	+.79	+.41
12	+.74	-.14	+.30	+.77	+.33	+.55	-.90	-.69	-.79
13	+.66	+.85	+.75	+.61	+.65	+.63	-.37	+.56	+.09
14	+.20	+.75	+.47	+.83	+.74	+.78	-.65	-.72	-.68
15	-.23	+.90	+.33	-.23	+.88	+.32	-.02	+.82	+.40
16	+.90	-.33	+.28	+.96	-.23	+.36	-.05	-.44	-.24
17	+.83	+.94	+.88	+.87	+.95	+.91	-.62	-.04	-.33
18	-.39	-.98	-.68	+.64	-.19	+.22	-.59	-.98	-.78
19	+.85	+.96	+.90	+.90	+.99	+.94	-.44	-.96	-.70
20	+.09	+.69	+.39	+.13	+.95	+.54	-.21	-.97	-.59
Average			+.49			+.59			+.01
Average	+.34	+.65	+.49	+.48	+.71	+.60	-.12	+.15	+.01

Bibliography.

Arai, Tsuru, 1912. Mental Fatigue

Teachers College Columbia University, Contribution to Education, No. 54.

Browne, C. E., 1906. The Psychology of the Simple Arithmetical Processes.

Am. Jour. Psych. Vol. 17. pp 1 - 37.

Courtis, S. A., 1913. Report of the Courtis Tests in Arithmetic.

Interim Report, Committee on School Inquiry, Board of Estimate and Apportionment, N. Y.

Courtis, S. A., 1911. Standard Tests in Arithmetic.

Elementary School Teacher, Vol. XII, No. 3.

Fox, W. A., and Thorndike, E. L., 1902. a - Relationships between the Different Abilities Involved in the Study of Arithmetic.

Columbia University Contributions to Philosophy, Psychology, and Education. Vol. XI, No. 1.

b - Sex Differences in Arithmetical Ability.

Columbia University Contributions to Philosophy, Psychology and Education. Vol. XI, No. 1.

Heck, W. H., 1913. a-A Study of Mental Fatigue in Relation to the Daily School Program.

J. H. Bell Co.

b - A Second Study of Mental Fatigue in Relation to the Daily School Program.

Psychological Clinic, Vol. VII, No. 2. pp. 29 - 34.

Rice, J. M., 1902. Educational Research: A Test in Arithmetic.

The Forum, Vol. 34.

Stone, C. W., 1908. Arithmetical Abilities and Some Factors Determining Them.
Columbia University Contributions to Education, Teachers College Series, No. 19.

Thorndike, E. L., 1904. Mental and Social Measurements.
The Science Press.

Thorndike, E. L., 1905 - 6. Measurements of Twins.
Archives of Philosophy, Psychology and Scientific Methods, Vol. 1.

Thorndike, E. L., 1907. Empirical Studies in the Theory of Measurement.
Archives of Psychology, No. 3.

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